

Quantum fluctuations of classical skyrmions in Quantum Hall Ferromagnets

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In this article, we discuss the effect of the zero point quantum fluctuations to improve the results of the minimal field theory which has been applied to study skyrmions in the quantum Hall systems. Our calculation which is based on the semiclassical treatment of the quantum fluctuations, shows that the one-loop quantum correction provides more accurate results for the minimal field theory.

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The novel development of the semiconductor technology has led to interesting observations of topological objects, skyrmions, in two component quantum Hall systems¹ where the spin is considered as the additional degree of freedom in the single layer quantum wells. These recent experiments have provided a situation in which exotic topological field theories could be examined. These models which have been developed for the elementary particles within the high energy physics, have been generalized to make a reliable field theoretic model at the limit of low energy physics for condensed matter systems, e.g., quantum Hall ferromagnets (QHF). Among those theories, the novel idea of the generalized Skyrme model² has attracted the attention of several authors.³⁻⁷ It has been shown^{4,7} that the accuracy of this model is satisfactory when the Zeeman energy is small compared to the Coulomb energy between the electrons, $e^2/\epsilon\ell_0$ where ℓ_0 is the magnetic length and ϵ is the dielectric constant of the host semiconductor. Within this approximation, spin is mapped to a classical unit vector (spin coherent state), as an order parameter. This correspondence has been obtained within the framework of the classical field theory, where the appropriate mean field theory for the low energy quantum Hall effect (QHE) has been considered as the Chern-Simon-Landau-Ginsburg model (CSLG).³ Integrating out the charge fluctuations of the composite bosons yields a minimal non-linear σ (NL σ) model for the spin, generalized to include Coulomb and Zeeman interaction terms.^{4,5} This has led to observing the finite size skyrmion spin texture where the size of skyrmion is given by the competition between the Zeeman energy and the Coulomb energy. Recently, the range of validity of such minimal field theory description has been investigated by a comparison with microscopic Hartree-Fock (HF), exact diagonalization, and a proposed variational wave function by Abolfath *et al.*⁷ It has been shown that the minimal field theory description is accurate for skyrmions with large spin quantum numbers, K ($\gtrsim 10$) although, as expected, it fails quantitatively for the baby skyrmions ($K < 10$). The microscopic HF can be regarded as an effective classical mean field theory with the appropriate boundary conditions, which retains the higher order gradient terms (these terms are absent in

the minimal field theory). On the other hand, the zero point quantum fluctuations (ZPQF) which are present in the quantum models (exact diagonalization and the variational wave function), are absent in these classical mean field theories. The higher order gradient terms are needed to improve the results of the minimal field theory to match the microscopic HF level and the ZPQF's are necessary for the quantum corrections. Recently Moon and Mullen⁸ pointed out for baby skyrmions, i.e., those typical in GaAs samples¹, the charge-density interaction within the CSLG theory, changes from a Coulombic one $1/r$ to $\ln(1/r)$ at short distances. They have claimed that the minimal field theory with the logarithmic interaction yields a good quantitative agreement with microscopic study. In this article we report on a study of the one-loop quantum corrections of the charged skyrmions in QHF as an alternative approach. We suggest that the differences between the minimal field theory and the microscopic picture for the baby skyrmions can be reduced by including the ZPQF. For instance, the microscopic calculation shows that the absolute value of the z-component of the spin is less than unity at the center of the skyrmion. In contrast, within the classical mean field theories the spin is constrained to be down at the origin. In fact, this is one of the boundary conditions used to solve the Euler-Lagrange differential equation. Such discrepancy can not even be removed by including the infinite number of higher-order derivative terms to the minimal field theory and/or replacing the effective Coulomb interaction between skyrmions with the logarithmic interaction, the approach which has been reported by Moon and Mullen.⁸ The ZPQF modifies the shape of skyrmion, consistent with the microscopic pictures. These corrections which are imposed on the classical models, are present in any non-collinear spin system, e.g., antiferromagnets. The negative poles of their Green's function are responsible for the presence of the ZPQF. For baby skyrmions, the ZPQF is sufficiently severe that the modification of the classical solutions becomes significant. Conversely, we do not find any correction to the Belavin-Polyakov's solutions^{9,13} where the result of the minimal field theory is identical to the microscopic picture.⁷ In this case, the ZPQF is negligible

for large enough skyrmions (adult skyrmions). We finally discuss the importance of the higher order gradient terms. We start to take the fluctuations into account by making use of the functional integral approach within the gaussian level of approximation. Note that the skyrmion charge of a class of configurations can not be changed by local fluctuations. The possible way to keep the charge and/or winding number, $Q[\equiv \int d\mathbf{r}\rho(\mathbf{r})]$, invariant, is: relaxing the \mathbf{m}_z due to in-plane fluctuations even at the boundaries of the system. Therefore in the space of Q 's, ergodicity is broken and the path integration must be evaluated for a pure state characterized by a given Q . Let us consider the fluctuations around the mean field solution for a given Q -skyrmion where its functional integral is in the following form:

$$Z_Q = \oint \mathcal{D}\mathbf{m} \delta(\mathbf{m}^2 - 1) e^{-S[\mathbf{m}]/\hbar}. \quad (1)$$

S is the Euclidean action, $S = S_{WZ}[\Gamma] + S_H$. The first term is the usual Berry's phase of spin history over a closed orbit, Γ , on the unit sphere in the presence of a unit magnetic monopole at the center, i.e., the Wess-Zumino term¹² and $S_H \equiv \int_0^{\hbar\beta} d\tau E[\mathbf{m}]$ where

$$E[\mathbf{m}] = E_0[\mathbf{m}] + E_z[\mathbf{m}] + E_{\text{Coul}}[\mathbf{m}]. \quad (2)$$

$E_0[\mathbf{m}]$ is the leading order term in a gradient expansion of the energy functional, i.e., the conventional Non-linear (NL) σ -model, and $E_z[\mathbf{m}]$ is the Zeeman energy functional. For QHF where skyrmions are carrying electric charge, next to the leading term in the gradient expansion is the non-local Coulomb energy, $E_{\text{Coul}}[\mathbf{m}]$, where the appropriate charge density associated with this term is equivalent to the filling factor ν times the Pontryagin density.^{4,5} The static solution of the spin configuration, $\tilde{\mathbf{m}}$, satisfy a non-linear differential equation which can be obtained by minimizing the energy functional, Eq.(2), with respect to \mathbf{m} .⁷ To compute the quadratic fluctuations around this saddle point solution, we must diagonalize the quadratic action. Following Polyakov,^{9,10} we exploit the renormalization procedure via introducing a complex scalar field, $\psi(\mathbf{r}) = \phi_1(\mathbf{r}) + i\phi_2(\mathbf{r})$, where the fluctuation of \mathbf{m} around its minimal solution is parametrized by $\mathbf{m} = \sqrt{1 - \bar{\psi}\psi} \tilde{\mathbf{m}} + \sum_{a=1}^2 \phi_a \mathbf{e}^a$ and $\tilde{\mathbf{m}}(\mathbf{r}) \cdot \mathbf{e}^a(\mathbf{r}) = 0$. The $\tilde{\mathbf{m}}(\mathbf{r})$ defines a map between the compactified representation space and the order parameter space. It induces a metric and therefore a set of connections on the order parameter manifold. The connections may be interpreted as the geometrical Chern-Simon gauge field strength, $A_\mu \equiv -\mathbf{e}_1 \cdot \partial_\mu \mathbf{e}_2 / 2$. Therefore, the scalar complex field describes a system of charged bosonic quasi-particles in the presence of the geometrical gauge field.^{10,11} Here we consider a stationary spin-texture hence no time-dependent geometrical gauge field strength, i.e., a pure geometrical magnetic field. One may easily show that the geometrical-magnetic field is proportional to the spin-texture density (electron density)^{10,11}, e.g., $\nabla \wedge \mathbf{A}(\mathbf{r}) = \frac{2\pi}{\nu} \rho(\mathbf{r})$. This is the zeroth

component of the conserved current which is obtained as $J_\mu = (\nu/2\pi)\epsilon_{\mu\nu\lambda}\partial_\nu A_\lambda$ by Lee and Kane in the context of the CSLG theory.³ The topological charge of skyrmions, Q , gives the vorticity of the geometrical gauge field, $\phi_v = 2\pi Q/\nu$. Interchanging two of these vortices yields an Aharonov-Bohm phase factor, $\exp(i\pi Q/\nu)$. This correctly reproduces the original statistics of the skyrmions. We interpret the super-current of the vortices, J_μ , as the topological current of the skyrmions and/or the quantum Hall current. Now we may expand the action, up to the quadratic terms about its classical solution ($\psi_c = 0$) where $\delta\mathbf{m} = \phi_a \mathbf{e}^a - \bar{\psi}\psi \tilde{\mathbf{m}}/2 + \mathcal{O}(\phi^4)$ and

$$S_{\text{eff}}[\bar{\psi}, \psi] = \frac{1}{2} \int_0^{\hbar\beta} d\tau \int d\mathbf{r} (\bar{\psi} \psi) \hat{\mathcal{H}}_{\text{eff}} \left(\frac{\psi}{\bar{\psi}} \right), \quad (3)$$

where

$$\hat{\mathcal{H}}_{\text{eff}} \equiv \frac{1}{2} \begin{pmatrix} \frac{\hbar}{8\pi\ell_0^2} \frac{\partial}{\partial\tau} + \hat{H}(\mathbf{r}) & U(\mathbf{r}) \\ U^*(\mathbf{r}) & \frac{-\hbar}{8\pi\ell_0^2} \frac{\partial}{\partial\tau} + \hat{H}^\dagger(\mathbf{r}) \end{pmatrix}. \quad (4)$$

In Eq.(4), the imaginary time derivative term is the the quadratic Wess-Zumino term and $\hat{H}(\mathbf{r}) = \rho_s(\frac{1}{i}\nabla - 2\mathbf{A})^2 + V(\mathbf{r})$ is a second order Hermitian operator and ρ_s is the spin stiffness. The effective Hamiltonian, $\hat{\mathcal{H}}_{\text{eff}}$ has two zero eigenvalues corresponding to global rotation and translation.¹⁴ One has to treat the zero modes properly by making use of the collective coordinates.¹³ To evaluate $\hat{H}(\mathbf{r})$, we exploit $\partial_\mu A_\mu = 0$, associated with the appropriate choice of the Coulomb gauge. A modified form of $\hat{\mathcal{H}}_{\text{eff}}$ for the simple NL σ model has been presented in Ref. 11. The off diagonal terms in \mathcal{H}_{eff} are the remnant of the Mott-insulating gap for the bosonic fields, ψ , where the vortices superconducting through the system. This can be interpreted as a dual picture for the quantum Hall state where the Mott-insulating gap of the bosons and the super-current of the vortices are equivalent to the quantum Hall gap and the quantum Hall current, respectively. Recently, this duality has been applied to investigate the phase diagram of the many-skyrmionic quantum Hall systems.¹⁶ Armed with the second functional derivative form of action, one may eventually evaluate the contribution of the one-loop quantum correction to the classical solutions of the quantum Hall NL σ -model. The rest of this paper is devoted to the study of the local fluctuations associated with a single skyrmion ($Q = \pm 1$) where its classical solution exhibits the circular symmetry and $A_\phi = (1 + \tilde{\mathbf{m}}_z)/2r$. One may expand the complex field, $\psi(\mathbf{r}, \tau)$, in terms of the eigenstates of the \hat{H} and its temperature dependent Fourier transform

$$\psi(\mathbf{r}, \tau) = \sum_{n, \Omega, m} C_{n, \Omega, m} f_{\Omega, m}(r) e^{im\varphi} e^{-i\omega_n \tau}, \quad (5)$$

where $\omega_n = 2\pi n/\beta$ are the bosonic Matsubara frequencies, $f_{\Omega, m}(r) = \langle r | \Omega, m \rangle$ are the orthonormal radial eigenfunctions, $\hat{H}|\Omega, m\rangle = \varepsilon_{\Omega, m}|\Omega, m\rangle$, $\varepsilon_{\Omega, m}$ is real and

$\langle \Omega', m' | \Omega, m \rangle = \delta_{\Omega', \Omega} \delta_{m', m}$. After a little algebraic manipulation, the effective action can be obtained

$$S_{\text{eff}}[\bar{C}, C] = \frac{\hbar\beta}{4} \sum_n \sum_{\Omega, m} \times (\bar{C}_{n, \Omega, m} C_{-n, \Omega, -m}) \mathcal{S}_{n, \Omega, m} \left(\frac{C_{n, \Omega, m}}{\bar{C}_{-n, \Omega, -m}} \right), \quad (6)$$

where

$$\mathcal{S}_{n, \Omega, m} \equiv \begin{pmatrix} \frac{-i\hbar\omega_n}{8\pi} + \varepsilon_{\Omega, m} & U_{\Omega, m} \\ U_{\Omega, m}^* & \frac{i\hbar\omega_n}{8\pi} + \varepsilon_{\Omega, -m} \end{pmatrix}. \quad (7)$$

Here the scale of length is $\ell_0 = 1$ and $\omega_{-n} = -\omega_n$. The contribution of the one-loop quantum corrections to the classical ground state energy, may be obtained by integrating out the fluctuations \bar{C} , and C . It leads to $E[\mathbf{m}] = E_c[\mathbf{m}] + E_f$ where $E_c[\mathbf{m}]$ is the energy of the classical skyrmion and

$$E_f = \lim_{\beta \rightarrow \infty} \frac{1}{\beta} \sum_n \sum_{\Omega, m} \ln \left\{ \left(\frac{\beta}{4} \right)^2 \left[- \left(\frac{i\hbar\omega_n}{8\pi} \right)^2 + \frac{i\hbar\omega_n}{8\pi} \times (\varepsilon_{\Omega, m} - \varepsilon_{\Omega, -m}) + \varepsilon_{\Omega, m} \varepsilon_{\Omega, -m} - |U_{\Omega, m}|^2 \right] \right\}. \quad (8)$$

We perform the Matsubara sum in the usual way to obtain E_f . It turns out an expression for E_f , i.e., a sum over the Bose-Einstein distribution functions, $n_B(\pm\beta\omega_{\Omega, m}^\alpha)$. Here $\omega_{\Omega, m}^\pm = 4\pi(\varepsilon_{\Omega, m} - \varepsilon_{\Omega, -m} \pm \sqrt{(\varepsilon_{\Omega, m} + \varepsilon_{\Omega, -m})^2 - 4|U_{\Omega, m}|^2})$ are the poles of the action, \mathcal{S} , and α stands for either positive or negative sign. At zero temperature, $n_B(x)$ is zero or -1 for positive or negative x 's. Therefore, the problem of evaluating the fluctuations at zero temperature is converted to the problem of finding the negative poles of \mathcal{S} . For the ferromagnet ground state where all spins are lined up, $U = 0$ and the eigenvalues are continuous and positive ($\omega^\pm \propto k^2$) hence no ZPQF. However, for the non-collinear spin texture namely the antiferromagnets, both negative and positive poles ($\omega^\pm \propto \pm k$) are present. At zero temperature, the negative poles contribute to ZPQF, and giving the correct shape of the ground state in agreement with the standard result of the Holstein-Primakov transformation.¹⁵ Note that in any case, \mathcal{S} is positive definite to guarantee that the classical solutions are the real minima of the action. To see this, let us set ω_n to zero and diagonalize the matrix in Eq.(7). We find that eigenvalues are positive. In order to estimate the effect of the ZPQF on the classical skyrmionic solution and then the enhancement on its energy, we evaluate the spectrum of the \mathcal{S} . The numerical calculation shows that the $m = 0$ is the most significant channel which contributes to ZPQF at $T = 0$. The effective potential of the $\hat{H}(\mathbf{r})$, contains two terms when the angular momentum, m is non-zero. A linear term, mA_ϕ/r and a quadratic term, m^2/r^2 . Near the core of skyrmion, the linear term is negligible in comparison with the quadratic term, since $A_\phi \sim r$. Approximately, we have $\varepsilon_{\Omega, m} = \varepsilon_{\Omega, -m}$ then

$\omega_{\Omega, m}^\pm = \pm 8\pi\sqrt{\varepsilon_{\Omega, m}^2 - |U_{\Omega, m}|^2}$. In this case, the $\omega_{\Omega, m}^+$ is positive and there will be no contribution to ZPQF at $T = 0$. We find:

$$E_f = 8\pi \sum_{\Omega, m} \sqrt{\varepsilon_{\Omega, m}^2 - |U_{\Omega, m}|^2}. \quad (9)$$

The effect of the fluctuations is to increase the energy cost of the skyrmions. E_f (which is proportional to the energy gap) is positive and decreases as the size of the skyrmion is increased, i.e., $E_f(K) > E_f(K+1)$. The energy differences between a skyrmion with K and $K+1$ spin-flips, $\Delta(K)$, estimates the level crossing between two skyrmions, as pointed out by Abolfath *et al.*⁷ One may note that $\Delta(K)$ can be renormalized by the ZPQF

$$\Delta(K) = \Delta_c(K) + \Delta_f(K), \quad (10)$$

where $\Delta_f(K) = E_f(K) - E_f(K+1)$, is the contribution of the ZPQF on $\Delta_c(K)$, the bare energy differences.⁷ One may also show that the value of the Zeeman splitting factor $(2t)$ corresponds to $\Delta(K)$. Then it is renormalized via ZPQF ($t \rightarrow t^*$). We note that $\Delta_f(K)$ vanishes rapidly as K increases, when the minimal field theory matches the microscopic results. We conclude that the use of the leading gradient terms in the minimal field theory is seriously in error for small K and the ZPQF can not even give a better prediction for $\Delta(K)$. This reconfirms the previous results of Ref. 7. Conversely, including the ZPQF can improve the shape of the skyrmions, and their z -component of the classical solution, $\tilde{\mathbf{m}}_z(\mathbf{r})$. In the following we study the effect of the ZPQF on $\mathbf{m}_z(r)$ via exploiting the above techniques to find out the correct shape of the skyrmions. It can be taken into account through

$$\mathbf{m}_z(\mathbf{r}) = \tilde{\mathbf{m}}_z(\mathbf{r}) \sqrt{1 - \bar{\psi}(\mathbf{r})\psi(\mathbf{r})} + \sum_{a=1}^2 \phi_a(\mathbf{r}) \hat{z} \cdot \mathbf{e}^a(\mathbf{r}). \quad (11)$$

We may evaluate the effect of the quantum fluctuations on \mathbf{m}_z via the standard technique of gaussian integrals and power series expansion of $\sqrt{1 - \bar{\psi}\psi}$. Note that the last term in Eq.(11) vanishes after integration. Defining the single point correlation function

$$G(\mathbf{r}) = \langle \bar{\psi}(\mathbf{r})\psi(\mathbf{r}) \rangle \equiv \lim_{\tau \rightarrow 0^-} \langle T_\tau \psi(\mathbf{r}, \tau) \bar{\psi}(\mathbf{r}, 0) \rangle, \quad (12)$$

where the ensemble average is denoted by $\langle \dots \rangle$ and T_τ is the time ordering operator and taking advantage of the Wick's theorem turn out the expectation value of the spin's z -component in terms of the $G(\mathbf{r})$

$$\langle \mathbf{m}_z(\mathbf{r}) \rangle = \left(1 - \frac{1}{2}G(\mathbf{r}) + \frac{1}{4}G^2(\mathbf{r}) \right) \tilde{\mathbf{m}}_z(\mathbf{r}) + \mathcal{O}(G^3). \quad (13)$$

We can evaluate $G(\mathbf{r})$ after expanding ψ in terms of the $f_{\Omega, m}(r)$'s. Integrating out the fluctuations and summing upon the Matsubara frequencies at zero temperature leads to

FIG. 1. The radial distribution of spin, m_z , is plotted. The effect of the ZPQF (solid line) on the classical distribution of spin (dashed line) is demonstrated for the $K = 1$ skyrmion. For comparison, the solution of the microscopic HF picture (dotted line) is plotted. The absolute value of m_z at the origin is suppressed by ZPQF. In the inset, the dashed line and the dotted line show the effective potential, $V(r)$ (ρ_s), and its bound state (WF), for the $m = 0$ channel, respectively.

$$G(\mathbf{r}) = -32 \sum_{\Omega, m} f_{\Omega, m}^2(r) \left(1 - \frac{\varepsilon_{\Omega, m}}{\sqrt{\varepsilon_{\Omega, m}^2 - |U_{\Omega, m}|^2}} \right). \quad (14)$$

Note that $\hat{H}(\mathbf{r})$ depends on the shape of the background spin texture, e.g., the size of skyrmion. For non-zero m , the term like $1/r^2$ in $\hat{H}(r)$ smears its dependence on the shape of the skyrmion. The magnitude of the eigen-energies for the non-zero angular momentums is large enough that one may neglect their contribution in Eq.(14), i.e., the $m \neq 0$ channels do not change the magnitude of $\mathbf{m}_z(r)$ significantly. The $m = 0$ potential, and its unique bound state ($\Omega = 1$) is shown in the inset of Fig. 1. Our choice for the boundary condition is $df(r)/dr = 0$ at the origin. The curvature of the, $V(\mathbf{r})$, is reduced by increasing the size of skyrmion. For the smaller skyrmions, the depth of the potential becomes more negative. The radial distribution of the spin for the $K = 1$ single skyrmion is shown in Fig. 1. As one may see, the effect of the ZPQF is severe at the center of the skyrmion and decays at large distances. As we have expected on general grounds, the magnitude of the spin at the center becomes closer to the predicted in the microscopic picture.

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